Generalized Random Mobility Models for Wireless Ad Hoc Networks

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What is an wireless ad hoc network ?

- A collection of mobile nodes equipped with wireless communications and networking capabilities.
  - Vehicular ad hoc network is a kind of ad hoc network in which vehicles constitute the mobile nodes in the network.
- Devices can communicate with another node that is within their radio range (peer-to-peer communication) or one that is outside their radio range (multi-hop communication) using other terminals to relay packets or via fixed gateways.

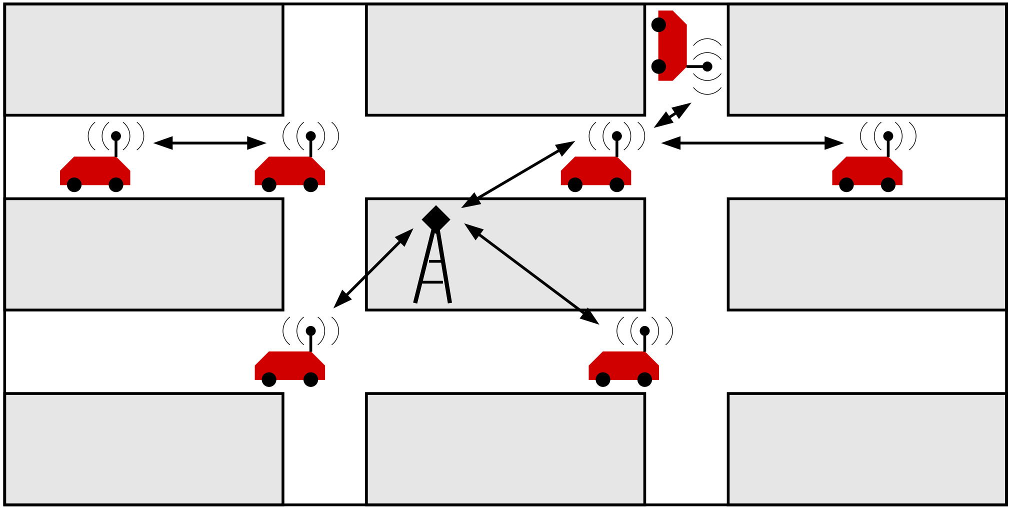


Figure 1: Multi-hop and peer-to-peer communications

Importance of Modeling Mobility

- Topology of the network continuously changes.
- Terminals can move according to different mobility profiles.
- In vehicular ad hoc networks, mobiles move with respect to highly dynamic mobility patterns, which result in highly dynamic network topologies.
- Mobility affects the performance of the protocols at different layers of the network.
- Mobility models that dictate the movement behavior of terminals play a key role in analyzing the impact of frequently changing topology on the performance of various protocol layers of these networks, which can be done by
  - simulation based studies, or analytical frameworks.
- How to model mobility in a realistic and simple way ?
  - For vehicular ad hoc networks, mobility model should be at least capable of differentiating highly and slowly moving terminals.

Classification of Mobility Models (1/2)

- Mobility models governs the changes in moving speed and direction of terminals according to a
  - random process, or a deterministic process.
- Example: Random Waypoint.
  - At the beginning of each movement epoch, terminal chooses a destination within the region with equal probability.
  - Moves to that destination with a speed from a given distribution, and pauses at the destination for a random amount of time.

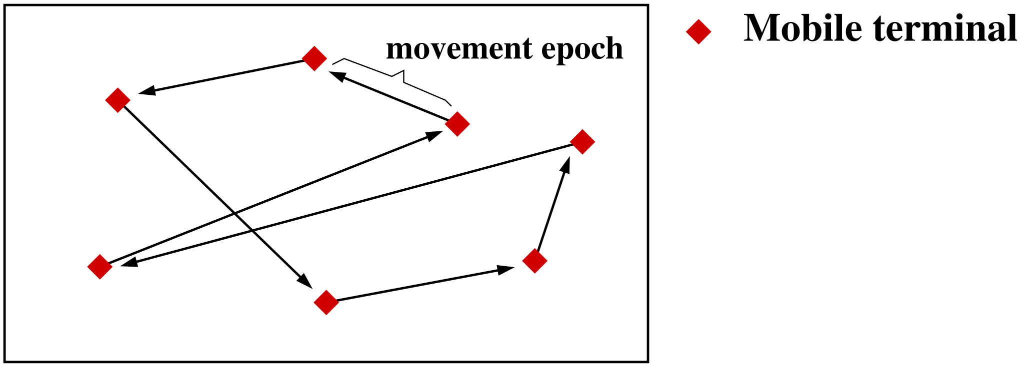


Figure 2: Movement pattern for Random Waypoint.

Classification of Mobility Models (2/2)

- It is important to use a realistic mobility model so that the result of the simulation or analytical study indicate the real-world performance of the system.
- Random Waypoint like models produce unrealistic patterns like sudden stops and sharp turns.
  - Since the choice of speed is not correlated with the distance that is going to be traveled a vehicle may travel long distances with unrealistically low speeds. As a result, the convergence time of simulation will increase.
- Not suitable to model scenarios where vehicles accumulate around hotspots.

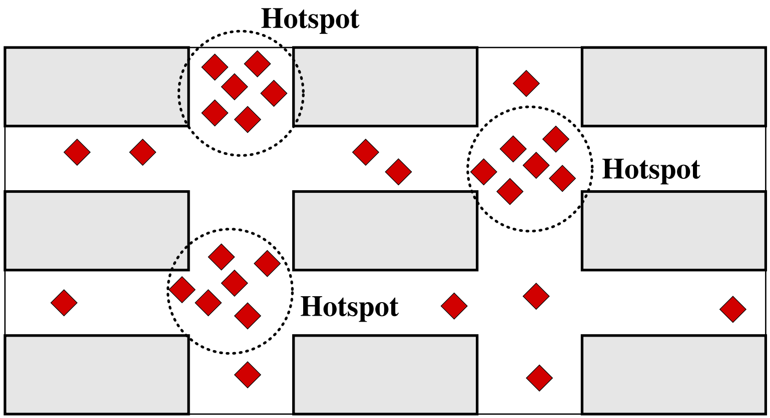


Figure 3: Accumulating mobile terminals around hotspots.

Importance of Long-Run Analysis

- For a given mobility model, the study of the long-run node spatial distribution is important in number of ways. It is required to evaluate:
  - link distance distribution between terminals,
  - route length distribution,
  - the throughput between source to destination pairs,
  - capacity of asynchronous MAC layer protocols.
    - Assuming the spatial load distribution to be uniform may not be valid for different mobility scenarios.
- For the simulation based studies, the values of the performance measures of interest can be only gathered after the speed and location distributions converge to their stationary values.
  - Hence, to reach consistent results from simulation we need to know the stationary distributions of location and speed.
- Simplicity: Is it possible to present long-run location and speed distributions with analytical expressions ? If possible, in closed form.

Generalized Random Mobility Model (1/3)

- Mobility pattern of a terminal is composed of consecutive movement epochs between starting (i.e., X_s) and destination (i.e., X_d) points.
- Each movement epoch is formulated as follows:
  - Terminal located at the point X_s selects X_d as destination according to conditional pdf $f_{X_d|X_s}$.
  - Moves to X_d with a speed V that is drawn from $[v_{\min}, v_{\max}]$ according to the conditional pdf $f_{V|X_s,X_d}$.
  - Pauses at X_d for T_p secs that is determined from conditional pdf $f_{T_p|X_d}$.

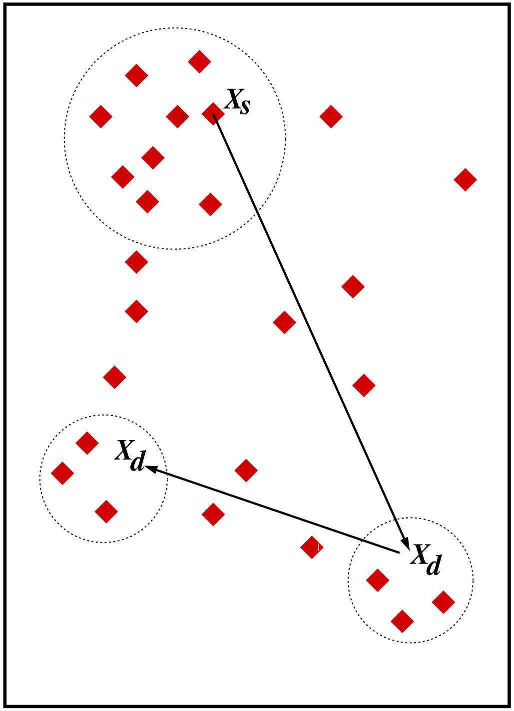


Figure 4: Movement pattern

Generalized Random Mobility Model (2/3)

- Movement pattern of terminals is characterized by the triplet: $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$.

$f_{X_d|X_s}$:
- define hotspots on the mobility terrain where mobiles accumulate with higher probability,
- and correlations between consecutive hotspot decisions can be successfully modeled.
  - For instance, the probability of selecting a hotspot as destination can be different from different starting points.

- Ex:

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} p_{1,2}\ g_2(x_s, x_d), & \text{if } x_s \in H_1 \text{ and } x_d \in H_2 \\ p_{2,1}\ g_1(x_s, x_d), & \text{if } x_s \in H_2 \text{ and } x_d \in H_1 \\ \ldots\ldots, & \end{cases}$$

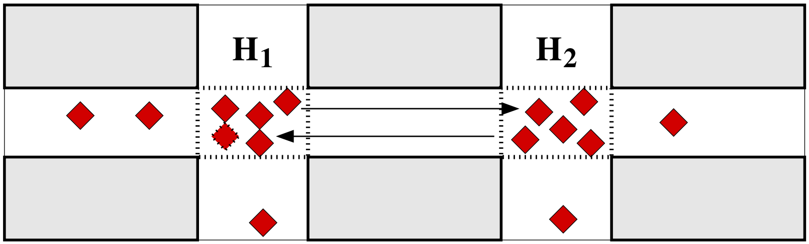


Figure 5: Simple scenario including hotspots.

Generalized Random Mobility Model (3/3)

$f_{V|X_s,X_d}$: Gives the flexibility of constructing a correlation between the distribution of V and the locations of the starting point X_s and destination X_d.

- For instance, a scenario that identifies V proportional to $|X_s - X_d|$, can be easily defined.
  - Otherwise, mobiles may "stuck" traveling long distances at low speeds and convergence time of simulation will increase.
- Can even be used to capture different *acceleration* characteristics of vehicles.
- We can model a case where the speed choices in the specific parts of mobility terrain can be restricted by the speed limits.

$f_{T_p|X_d}$: It makes it possible to capture different pause distributions at different destinations available for the mobility model.

Analytical Framework (1/2)

- Let $\mathbf{X}(t)$ denote the state of the mobile terminal at time t.
- The stochastic process $\{\mathbf{X}(t), t \geq 0\}$ must be defined on a continuous state space that has separate dimensions for current location and destination coordinates, and speed.
- We built our solution according to the following assumptions:

A_1 : The mobility terrain R is discretisized into n cells of the same shape, denoted by c_i. Movement epochs occur between two randomly picked starting and destination cells.

A_2 : Approximate V by a discrete random variable V^* taking values in the state space $\{z_1, z_2, \ldots, z_m\}$, where $z_1 \leq v_{\min}$ and $v_{\max} \leq z_m$.

A_3 : We observe the system at the embedded points T_k, $k \in \mathbf{N}$ in time that corresponds to one of the following events:

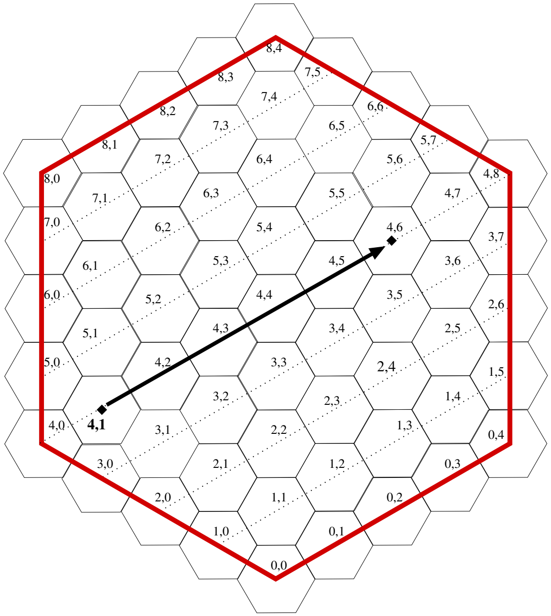
E_1 : Terminal starts moving towards target cell.

E_2 : Terminal enters to a different cell while moving.

E_3 : Terminal reaches to the destination cell.

Analytical Framework (2/2)

- The mobility formulation that is constructed according to the assumptions A_1, A_2, and A_3 is called as discretisized mobility formulation.
  - n and m are the discretization parameters. As $n \to \infty$ and $m \to \infty$ we converge to the original continuous case.



- Let $\mathbf{S}_k$, $k \in \mathbf{N}$, denote the state of the mobile terminal at time T_k. The state space of $\mathbf{S}_k$ will be defined as follows:

$$\begin{aligned} \mathcal{S} &= \{(c_i, c_j, z_r, q) \mid i, j = 0, \ldots, n-1, i \neq j, \\ &\qquad r = 1, \ldots, m, q = 1\} \\ &\cup \{(c_i, q) \mid i = 0, \ldots, n-1, q = 0\} \end{aligned}$$

- The stochastic process $\{\mathbf{X}(t), t \geq 0\}$ will be redefined on $\mathcal{S}$ by

$$\mathbf{X}(t) = \mathbf{S}_k, \quad \text{if } T_k \leq t < T_{k+1}$$

Results

- Let X and $\tilde{V}$ denote the random variables having the long-run distribution of location and speed, respectively.
- Let $f_{\tilde{V}}$ denote the pdf of $\tilde{V}$, and let $F_X(x, \Delta x_1, \Delta x_2)$ denote pmf of X over $R(x, \Delta x_1, \Delta x_2) = [x_1 - \frac{\Delta x_1}{2}, x_1 + \frac{\Delta x_1}{2}] \times [x_2 - \frac{\Delta x_2}{2}, x_2 + \frac{\Delta x_2}{2}]$, where $x = (x_1, x_2)$.

$$F_X(x, \Delta x_1, \Delta x_2) = \frac{E[T_p | X_s \in R(x, \Delta x_1, \Delta x_2)] \Pr\{X_s \in R(x, \Delta x_1, \Delta x_2)\}}{E[T_p | X_s \in R] + \hat{D}} + \frac{\int_{v_{\min}}^{v_{\max}} K(x, v, \Delta x_1, \Delta x_2)\, dv}{E[T_p | X_s \in R] + \hat{D}},$$

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p | X_s \in R] \delta(\tilde{v})}{E[T_p | X_s \in R] + \hat{D}}, & \tilde{v} = 0 \\ \frac{\sum_{R(x, \Delta x_1, \Delta x_2) \in \mathcal{S}(\Delta x_1, \Delta x_2)} K(x, \tilde{v}, \Delta x_1, \Delta x_2)}{E[T_p | X_s \in R] + \hat{D}}, & \tilde{v} \in [v_{\min}, v_{\max}] \end{cases}$$

- We designed a tool that numerically evaluates $F_X(x, \Delta x_1, \Delta x_2)$ and $f_{\tilde{V}}$ for the mobility characterizations done according to the triplet $< f_{X_d | X_s}, f_{V | X_s, X_d}, f_{T_p | X_d} >$.

- Ex: Random waypoint over $R = [0, a_1] \times [0, a_2]$. The parameters:

$$f_{X_d|X_s} = \frac{1}{a_1 a_2}, \quad f_{V|X_s,X_d} = \frac{1}{v_{\max} - v_{\min}}, \quad f_{T_p|X_d} = h(t_p)$$

- We obtained:

$$F_X(x, \Delta x_1, \Delta x_2) = \frac{\frac{E[T_p]\Delta x_1 \, \Delta x_2}{a_1 \, a_2} + E[\frac{1}{V}] \;\; K_X(x, \Delta x_1, \Delta x_2)}{E[T_p] + E[\frac{1}{V}]\bar{D}}$$

- There is no closed form expression for $F_X(x, \Delta x_1, \Delta x_2)$. However, we reached the following approximation:

$$\lim_{\substack{\Delta x_1 \to 0 \\ \Delta x_2 \to 0}} \frac{F_X(x, \Delta x_1, \Delta x_2)}{\Delta x_1 \, \Delta x_2} \approx \tilde{f}_X(x),$$

where

$$\tilde{f}_X(x) = \frac{E[T_p]\frac{1}{a_1 \, a_2} + E[\frac{1}{V}]k(x)/\tilde{N}}{E[T_p] + E[\frac{1}{V}]\bar{D}},$$

where $k(x)$ has a closed form expression.

  - We designed a simulation tool that captures all of the generalization provided by the model and observed that the approximation is highly accurate and
    - insensitive to the proportion between a_1 and a_2.

Consider an improvement for the random waypoint:

$$f_{V|X_s,X_d}(v|x_s,x_d) = \frac{Z\left(\frac{v-\mu(x_s,x_d)}{\sigma}\right)}{\sigma\left(\Phi\left(\frac{v_{\max}-\mu(x_s,x_d)}{\sigma}\right) - \Phi\left(\frac{v_{\min}-\mu(x_s,x_d)}{\sigma}\right)\right)},$$

where $\sigma > 0$, $v_{\min} \leq v \leq v_{\max}$, $\mu(x_s,x_d) = v_{\min} + \frac{(v_{\max}-v_{\min})}{\bar{D}}|x_s - x_d|$ (i.e. truncated normal distribution with parameters σ and $\mu(x_s,x_d)$).

- As $\sigma \to 0$, the possibility of determining V directly proportional to $|X_s - X_d|$ increases. Also, as $\sigma \to \infty$, V becomes uniformly distributed in $[v_{\min}, v_{\max}]$.
- Using our tool we evaluated $E[\tilde{V}]$ numerically.

$E[\tilde{V}]$ (m/s)

($v_{\min}=1$ m/s, $v_{\max}=20$ m/s)

$E[T_p]$ (sec)	$\sigma \to \infty$	$\sigma = 10$	$\sigma = 5$	$\sigma = 1$
0	6.342	6.517	6.867	8.106
15	5.408	5.985	6.279	7.299
30	4.713	5.535	5.785	6.638

- If there is a correlation between the choices of V and $|X_s - X_d|$, the value of $E[\tilde{V}]$ will also be affected by how destinations are chosen.

Conclusions

- We proposed a generalized mobility modeling approach and provided an analytical framework for its long-run analysis.
- Obtaining closed form expressions for long-run location and speed distributions is dependent on the choice of the parameters: $f_{X_d|X_s}$, $f_{V|X_s,X_d}$, and $f_{T_p|X_d}$.
- In the following paper (reference [1]) we also concentrated on the effects of the acceleration-deceleration characteristics of vehicles on the long-run measures of this generalized mobility modeling approach.

[1] Denizhan N. Alparslan and Khosrow Sohraby, "*A Generalized Random Mobility Model for Wireless Ad Hoc Networks and its Analysis: One-Dimensional Case*", Technical Report, University of Missouri-Kansas City, School of Computing and Engineering, htttp://d.web.umkc.edu/dna5a0.